Exp Brain Res (2003) 150:95–108
DOI 10.1007/s00221-003-1402-z

RESEARCH ARTICLE

Denise Y. P. Henriques · John F. Soechting

Bias and sensitivity in the haptic perception of geometry

Received: 19 September 2002 / Accepted: 16 January 2003 / Published online: 8 March 2003

Abstract Our ability to recognize and manipulate objects relies on our haptic sense of the objects' geometry. But little is known about the acuity of haptic perception compared to other senses like sight and hearing. Here, we determined how accurately humans could sense various geometric features of objects across the workspace. Subjects gripped the handle of a robot arm which was programmed to keep the hand inside a planar region with straight or curved boundaries. With eyes closed, subjects moved the manipulandum along this virtual wall and judged its curvature or direction. We mapped their sensitivity in different parts of the workspace. We also tested subjects' ability to discriminate between boundaries with different degrees of curvature, to sense the rate of change of curvature, and to detect the elongation or flattening of ellipses. We found that subjects' estimates of the curvature of their hand path were close to veridical, and did not change across the workspace though they did vary somewhat with hand path direction. Subjects were less accurate at judging the direction of the hand path in an egocentric frame of reference, and were slightly poorer at discriminating between arcs of different curvature than at detecting absolute curvature. They also consistently mistook flattened ellipses and paths of decreasing curvature (inward spirals) for circles – and mistook arcs of true circles for arcs of tall ellipses or outward spirals. Nevertheless, the sensitivity of haptic perception compared well with that of spatial vision in other studies. Furthermore, subjects detected curvature and directional deviations much smaller than those that actually arise for most reaching movements. These findings suggest that our haptic sense is acute enough to guide and train motor systems and to form accurate representations of shapes.

This work was supported by NIH Grant NS 15018. D.Y.P.H. was supported by a CIHR fellowship

D. Y. P. Henriques · J. F. Soechting
Department of Neuroscience,
University of Minnesota, Minneapolis, MN 55455, USA

D. Y. P. Henriques (✉)
Department of Neuroscience,
University of Minnesota, 6-145 Jackson Hall,
321 Church St., Minneapolis, MN 55455, USA
e-mail: denise@umn.edu
Tel.: +1-612-6263146
Fax: +1-612-6265009

Keywords Arm · Hand path · Humans · Psychophysics · Curvature · Direction · Tactile · Kinesthesia

Introduction

Haptic cues provide a rich source of information about nearby objects. We can sense shapes and surface qualities such as compliance and texture not by touch alone, but by correlating tactile sensations with kinesthetic cues resulting from active, exploratory movements of the arm and hand. As this brief synopsis implies, haptic sense involves the integration of a variety of somatosensory afferent information with efferent signals, and, most likely, cognitive factors as well. The present study aims to define the acuity of haptic perception, by assessing how well subjects can sense simple geometric properties such as the direction, curvature, and rate of change of curvature of surface boundaries.

Most studies have used tasks involving only small movements of the fingers and wrist or the manipulation of objects immediately in front of the subject (e.g., Davidson 1972; Gordon and Morison 1982; Pont et al. 1998; Armstrong and Marks 1999; Fasse et al. 2000). In our study, as in real life, subjects explored objects using large multijoint arm movements over a large range of positions relative to the body. It is unknown whether haptic sensitivity varies across space and for different arm configurations.

Another gap in our knowledge concerns the veridicality of haptic perception. Most studies have looked at relative judgments – how well we can match shapes, or discriminate between similar sizes, curvatures or orientations (Pont et al. 1998; Gentaz et al. 2001; Kappers 1999, 2002; Voisin et al. 2002a, 2002b). Less attention has been paid to our absolute judgment of curvature and orientation

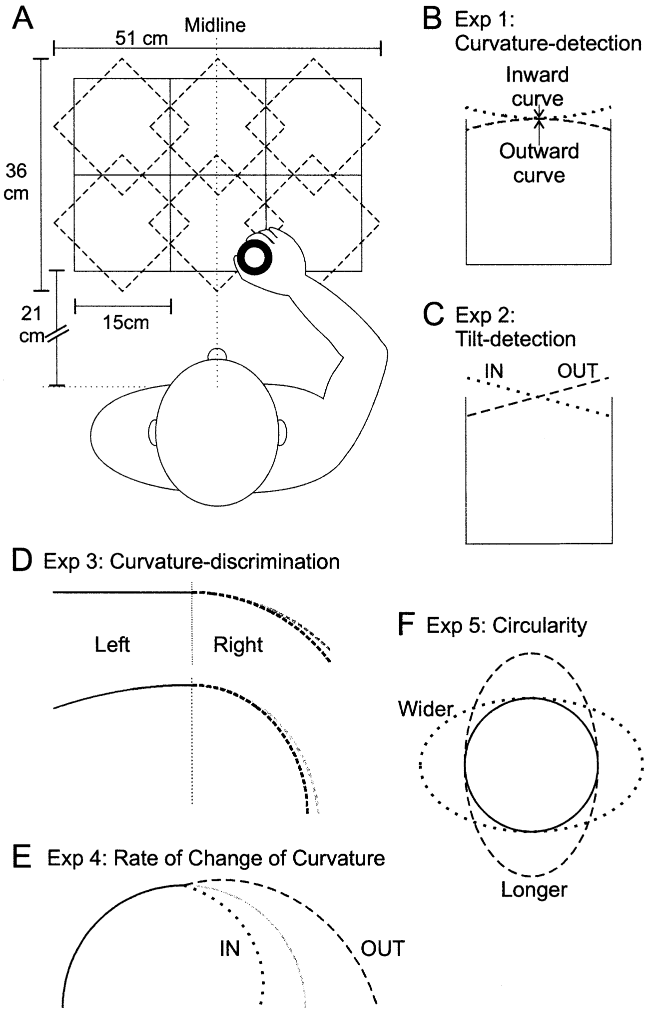

Fig. 1A–F Overview of the experiments. **A** The layout of the boundaries, generated by the manipulandum, along which subjects moved their hands when detecting the curvature or directional tilt of their resulting hand path. In the Curvature-detection experiment, subjects moved along 48 boundaries oriented in the cardinal directions (*solid lines forming a grid*) and 45° diagonally (*dashed lines*). In the Tilt experiment, only the cardinal boundaries were tested. The boundaries were placed so that they formed squares, each 15×15 cm in size. Subjects were tested on one boundary at a time, and made 2-AFC about the curvature or tilt of their hand trajectory. The shared boundaries were tested from both directions. **B** Experiment 1, Curvature-detection. Subjects judged whether their hand path had curved outwards (*dashed lines*) or inwards (*dotted lines*). For better visibility, the arcs are drawn with double-magnified curvature compared to those tested on the initial staircase trials (curvature of 2.0/m). **C** Experiment 2, Tilt. Subjects judged whether their hand path had tilted ‘in’ or ‘out’ with respect to a specified cardinal direction. Each staircase began with a straight-boundary tilting 15° CCW (out) or CW (in) with respect to the subject. **D** Experiment 3, Curvature-discrimination. Subjects indicated which arc – the left one or the right one – felt more curved. The sideways oriented arcs were joined so that the hand path was uninterrupted; a short beep told the subject when the hand crossed this transition point (*vertical dotted line*). In task one (*top*), the reference arc, shown on the left by the *solid line*, was always flat, and the arc that varied with the staircase began with a curvature of 6.7/m (*dark dashed line*). In task two (*bottom*), the reference arc always had a curvature of 2.5/m and the variable arc began with a curvature of 10.0/m (*dark dashed line*). *Gray dashed lines* show the direction and initial step size after a correct response. **E** Experiment 4, Spiral or Rate of Change of Curvature. Subjects judged whether their hand path spiraled in or spiraled out. The staircase began so that after the 90° spiral, the hand was displaced 6 cm closer or farther from the arc’s rotation center than the initial 10-cm radius. **F** Experiment 5, Circularity. Subjects judged whether the ellipse they traced with the manipulandum was larger along the forward-backward axis (longer ellipse, *dashed trace*) or along the sideways axis (wider ellipse, *dotted trace*). One of the axes was kept a constant size (20 cm for the Big Circle task, 10 cm for the Small Circle task), while the size of the other axis varied with the adaptive staircase, and began 67% larger. The *solid trace* is a perfect circle

with respect to the body. In our study, we compare relative and absolute haptic judgments of these geometric quantities.

To measure how accurately subjects could sense the curvature and other geometric properties of their unseen hand trajectory, we had subjects grip a robot manipulandum and move it toward and then along a motion boundary that had been programmed into the robot. One of the main advantages of simulating objects with a manipulandum was that we could quickly vary the parameters (e.g., the amount and direction of curvature) for each trial based on subjects’ past responses, and thus determine their sensitivities and biases with great precision.

We assessed subjects’ ability to detect the absolute curvature of their hand path and the absolute directions, relative to the body, of straight hand paths. That is, subjects compared their haptic perception to their cognitive sense of straight and of directions that are purely forward or sideways in an egocentric frame. We determined whether subjects’ sensitivities and biases varied with the direction or location of the hand path. We further tested how well subjects discriminated between boundaries of different curvature, and whether they were better able to detect these differences in curvature than absolute curvature. Finally, we tested subjects’ ability to detect rate of change of curvature, and compression or stretching of circles.

Materials and methods

Subjects

Subjects were right-handed and had no history of sensory, perceptual or motor disorders. Their numbers in different experiments ranged from six to nine. Three subjects performed all five experiments, while another 12 participated in one or more. All gave informed consent, and all procedures were approved by the Institutional Review Board of the University of Minnesota.

Equipment and procedure

We measured how well subjects sensed their hand trajectory without seeing it while they moved a manipulandum attached to a two-jointed robot arm (Interactive Motion Technologies). Subjects sat facing the workspace, and grasped the vertical handle of the manipulandum with the right hand (Fig. 1A). The robot was programmed to keep the hand inside a horizontal planar region, just above waist level, with four intersecting boundaries simulating a square container (15×15 cm). At the boundary, subjects felt the resistance of the manipulandum as if they were hitting a wall. The resistance was generated by a force perpendicular to the wall and proportional to the depth of penetration with a stiffness of 2 N/mm and a viscous damping of 5 N/(mm/s). No force was exerted when subjects were inside the boundaries. Encoders on the shafts of the two torque motors measured position and velocity. This information was converted to Cartesian coordinates of the workspace. The actual position was compared to the boundaries of the virtual wall to generate an elastic force. Whenever this force was nonzero, a viscous damping force was added for stability. The inertia of the manipulandum was very low, with moments of inertia of 0.0195 and 0.0037 kg.m^2 for shoulder and elbow links (23 and 20 cm in length) respectively. Shadmehr et al. (1993) have estimated the manipulandum's inertia "expressed in the end-point coordinates [is about one-tenth of] that of a typical human arm."

Along one boundary of this simulated square, the haptic wall assumed the contour being tested. With their eyes closed, subjects moved along this boundary and made two-alternative forced choice (2-AFC) judgments about the resulting hand path. Subjects felt the boundary for as long as they wanted, and we did not impose any other restrictions on the subjects' strategy. Typically, subjects moved to and fro along the boundary a few times, at a rate of about 1 Hz, before responding. The experimenter keyed in the responses for a total of 44 forced choices for each tested boundary. From these responses, we mapped out each subject's sensitivity function. From it, we calculated a bias and degree of certainty for each boundary (see Fig. 3B), using the binary logistic fit in SPSS (Statistical Package for the Social Sciences).

Experiment 1: curvature detection

In the Curvature-detection experiment, six subjects (three of whom were females) were asked to report whether the hand path curved inward or outward (dashed and dotted lines in Fig. 1B). Subjects were tested on each virtual wall through a set of 44 sequential trials in which the wall varied in curvature (as described below) but not location. Before each set of trials, they were shown the general location and direction of the virtual wall that they would be feeling. The subjects began each trial by placing the manipulandum handle in an area indicated by the experimenter to be within the square boundary where no forces were exerted. After they closed their eyes, they moved the handle toward the virtual wall and then to-and-fro along its boundary for several seconds, until they could report whether the wall curved inward or outward. Once the subject's response was keyed-in, the forces generating the wall were terminated and the trial ended. The subject then returned the handle toward the center of the simulated square and signaled to the experimenter to begin the next trial. At the completion of the 44-trial sequence, subjects were shown the location and direction of the next virtual wall being tested.

The curvature of the virtual wall was adjusted using a 2-AFC adaptive staircase algorithm, decreasing if the response was consistent with the previous one and increasing if it was not (Kesten 1958; see Treutwein 1995 for a review). Two such staircases, one beginning with inward-curved boundaries and the other with outward-curved ones, were randomly interwoven, and each began with an arc of curvature 2.0/m (radius of curvature of 0.5 m). An example is shown in Fig. 3A (squares for the inward one and triangles for the outward one). Each time the subject altered his/her response from inward to outward or from outward to inward along a particular staircase, the step reversed direction, and its size decreased. Reducing the step size after each reversal ensured that subjects were tested more frequently on curvatures closer to their sensitivity threshold. Depending on subjects' responses, the curvature in either staircase might approach zero or even cross zero (switching from outward to inward or inward to outward). If subjects respond consistently, the two staircases should converge toward the subject's curvature bias (the curvature at which subjects have an equal probability of reporting an inward or outward curvature). This design, of two interwoven 2-AFC adaptive staircases, was used for all five experiments in this study.

We tested subjects' sense of curvature in different parts of the workspace, and for virtual walls or boundaries oriented in different directions in the horizontal plane: sideways, backward-forward, and diagonally 45° clockwise and counterclockwise (positive- and negative- diagonals) with respect to the body. Subjects moved along a total of 48 curved boundaries or virtual walls as shown in Fig. 1A (represented as straight solid-line and dashed-line segments forming 12 squares). The sideways and backward-forward boundaries formed a grid of six squares, with a total of 24 edges (solid-lines in Fig. 1A). Because these 24 boundaries formed six adjacent squares, some boundaries were shared and these were tested twice, but approached from different sides (and during different sessions). The 12 backward-forward arcs were located 7.5 and 22.5 cm to the left and right of the subject's midline. The 12 sideways arcs were located 21, 36, and 51 cm in front of the subject's torso. The 24 diagonal boundaries (dashed lines in Fig. 1A) also formed six diamond-squares, each centered on one of the six cardinal-squares so that they overlapped slightly, but were evenly distributed across the workspace. Outward curvature meant that the arc was convex in the 12 to 5 o'clock directions for sideways, forward-backward, and diagonal arcs. Subjects were given a diagram illustrating the alternative choices for each direction (similar to those shown in Fig. 1B), which they could view after each trial. Because subjects correctly indicated the direction of curvature on the first trial of each staircase, we were satisfied that the choices were clearly illustrated.

The entire Curvature-detection experiment took 6 h, divided into six 1-h sessions on different days. The order of locations and directions tested was randomized for each subject.

Experiment 2: tilt

We use the term "tilt" to refer to the direction that the virtual wall diverged from a direction either parallel (sideways) or orthogonal (forward-backward) with the body's frontal plane. In this experiment, seven subjects (four females) reported whether the hand trajectory felt tilted clockwise (CW) or counterclockwise (CCW) (IN or OUT, respectively, in Fig. 1C) with regard to what they thought was a strictly forward-backward or sideways direction with respect to the body and table. A diagram illustrated the two choices. They moved the hand along 24 straight boundaries tilting away from cardinal directions at 24 different locations (solid lines in Fig. 1A). The two staircases began with the straight boundary tilted 15° CW and CCW from the cardinal direction. Again, subjects were tested twice on shared edges of adjacent squares from different sides in different sessions.

The Tilt experiment took three 1-h sessions on different days. The order of locations and directions tested was randomized for each subject.

Experiment 3: curvature discrimination

In the next two experiments, the Curvature-discrimination experiment (Fig. 1D) and the Spiral experiment (Fig. 1E), subjects were asked to compare two arcs, and to judge which had a greater curvature or whether curvature in one was increasing or decreasing in comparison with the reference arc. The two arcs were oriented sideways and joined side-by-side so that one arc was on the left and the other on the right of the subject's midline. The entire width of the two adjoining arcs was 24 cm (12 cm each side) from wall edge to wall edge. For these experiments, the arcs were located about 25 cm in front of the subject; with the robot exerting the resistive force toward the subject. Subjects took about 15 min to complete each experiment.

The Curvature-discrimination experiment consisted of two tasks (Fig. 1D) testing the subject's sensitivity to differences in curvature. Eight subjects (three females) reported which side, left or right, of an arc was more curved. A beep sounded whenever the subject moved his/her hand across the transition point between the two sides. On one side, chosen randomly by the computer, was the reference arc (solid lines in Fig. 1D): a flat edge for task one (top), and an arc with a constant curvature of 2.5/m (radius 0.4 m) for task two (bottom). On the other side, the variable curve (dark dashed lines) began with a greater curvature, 6.67/m for task one and 10.0/m for task two. This curvature was adjusted by a single adaptive staircase for subsequent responses. Each side (left and right) had its own staircase, so the variable arc could increase or decrease in curvature at different rates, depending on whether it was on the right or left side. If the subject correctly named the side that was most curved, the variable arc would become closer in curvature to the reference line/arc, so that the difference in curvatures drew nearer to zero.

Experiment 4: spiral

The Spiral experiment tested subjects' sensitivity to rate of change of curvature. As in Experiment 3, subjects were presented with two adjacent arcs. One was a quarter-circle, with a constant radius of curvature of 0.1 m (a curvature of 10.0/m; solid line in Fig. 1E). At the midline transition point to the other arc (in the sagittal plane of the torso), the radius of curvature began to increase or decrease at a constant rate with respect to arc length, so that the hand path spiraled in or out (dashed and dotted arcs in Fig. 1E) over a range of 90°. The equation for this logarithmic spiral (also known as the Bernoulli spiral) in polar coordinates is as follows:

$$r = a * \exp(sp * \text{theta}) \quad (1)$$

where r is the radius of curvature, a is the initial radius of 10 cm, theta is the angle of the hand with respect to the arc's rotational center in polar coordinates, and *sp*, for spiral, is the rate of change of the radius relative to the arc length. The experiment began with a value of sp=0.30 for the spiraling-out staircase, and sp=–0.30 for the spiraling-in staircase. Accordingly, after 90° of spiral, the radius was increased to 16.0 cm or reduced to 6.2 cm, respectively, compared to the 10-cm radius at the transition point. As the subject correctly named the direction of spiral, the rate of change of the radius of curvature fell closer and closer to zero, approaching an arc with a constant radius equal to that of the reference arc. The seven subjects (four females) performed the experiment twice: once when the constant-radius arc was on the left and once when it was on the right.

Experiment 5: circularity

The Circularity experiment also tested subjects' sensitivity to differences in curvature by having eight of them (two females) identify whether the major (larger) axis of a traced ellipse was the forward-backward axis (length) or along the medial-lateral axis (width) as shown by the dashed and dotted ellipses in Fig. 1F. For one adaptive staircase, the width of the ellipse was 20 cm for all trials and its length was two-thirds longer (33.3 cm) for the first trial, and varied with each subsequent response. The axes were reversed for the other staircase: length was always 20 cm and the width began at 33.3 cm. We call this the Circularity experiment because as subjects identified the larger axis, the difference in the length of the two axes became smaller, gradually resembling what subjects would haptically perceive as a circle. We also tested whether this percept of a haptic circle changed with size by having subjects repeat the experiment for circular traces half as big: the constant axes 10 cm long/wide and the varying orthogonal axes having initial values two-thirds larger.

Results

Experiment 1: curvature detection

In the Curvature-detection experiment we measured what subjects sensed to be a straight hand path in different locations and directions. Figures 2A and 3 demonstrate the experimental procedure. In Fig. 2A, we show the results from a typical trial, which began with the hand in the center of region bounded by virtual walls. The subject moved toward one edge of the boundary (curved outward in this example), and moved back and forth along it for about four cycles with a peak speed of about 25 cm/s at a rate of about 1 Hz. The exploratory movements were generally smooth, the sharp speed transients at the minima reflecting the subject's impact with the two bounding surfaces roughly perpendicular to the trajectory. Although we did not routinely record the hand trajectories in these experiments, the results illustrated in Fig. 2A are representative of the strategy employed on the task (see also Fig. 2B, illustrating results for a typical trial for Experiment 3).

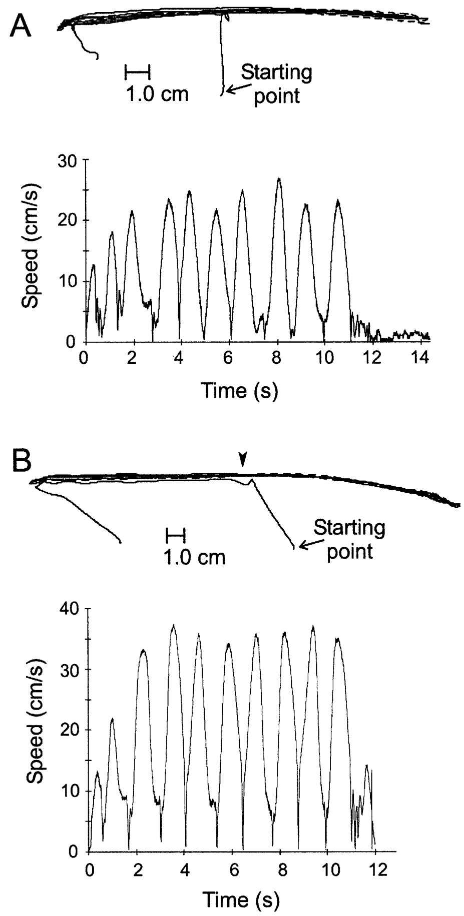

Fig. 2 Above view of the 2D trajectory of the manipulandum handle as one subject moved along the virtual curved wall for one trial in the Curvature-detection experiment **(A)** and for one trial in the Curvature-discrimination experiment **(B)**. *Solid-line trajectories* represent movement to the left while *dashed-line trajectories* represent movement to the right. Below the trajectory plot is the corresponding speed profile for the entire trial

Figure 3 illustrates how acuity of curvature detection was evaluated with an example from one subject who was asked to discriminate the inward/outward curvature in the forward-backward direction in the part of the workspace

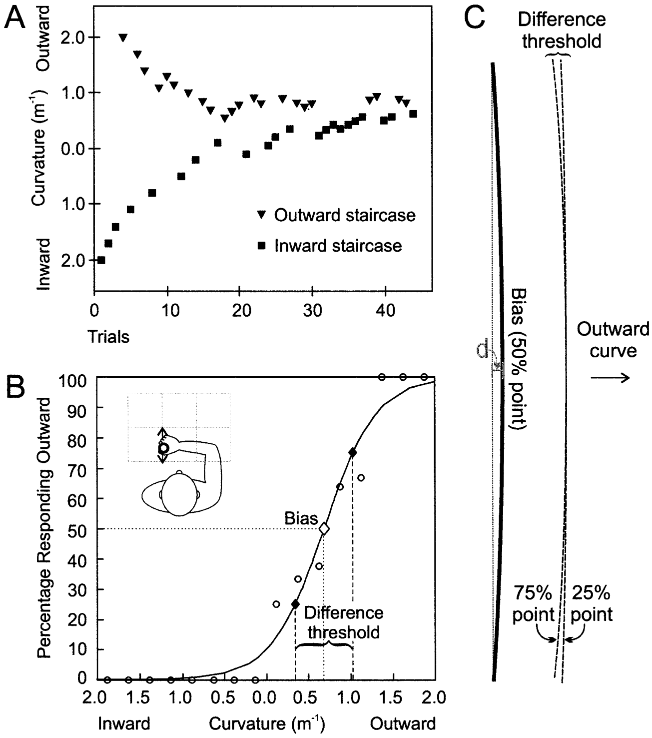

Fig. 3A–C Results for one subject detecting the curvature of a forward-backward boundary to his immediate left (see *inset in* **B**). **A** The sequence of boundary curvatures presented in each of the two adaptive staircases (*triangles* for the outward-curved staircase and *squares* for the inward one). **B** The sensitivity function fitted to these responses. *Open circles* mark the mean percentage by which the subject responded that his hand path was curved 'outward' for curvatures falling within each 0.25/m interval between 2.0/m inward and 2.0/m outward. The curvature bias (*open diamond*) is at the 50% point of the psychometric function, while the 25% and 75% points (*filled diamonds*) of the function mark the borders of the difference threshold (or uncertainty range). The bias for this subject was 0.69/m, with a difference threshold of 0.68/m. **C** This bias arc (*solid trace*) is drawn with respect to a veridical line (*dotted trace*). The *gray error bar* shows the peak hand displacement (*d*) of the curvature bias at the midpoint of the arc. The *dashed lines* on the right are the upper (75%) and lower (25%) difference threshold arcs

shown in the inset. Figure 3A shows how curvature varied over the 44 trials. The experiment started with two staircases (inward and outward) at an initial curvature of 2.0/m. If the subject's response was the same as his/her response in the previous trial on that particular staircase, curvature was decreased. If the response differed (e.g., response changed from inward to outward), the curvature was increased and the step size decreased. We fitted these responses to a standard psychometric function (with SPSS) for each hand path location (Fig. 3B). The 50% point of this sensitivity function is called the subject's curvature bias, because it represents the 'curvature' that the subject perceives equally likely as outward or inward. The curvature bias represents the curvature of the surface that a subject would perceive as straight or non-curving. We call it a bias because it represents the departure from a veridically straight wall that would have a curvature of zero. In this example, the bias was +0.69/m curvature (open diamond), i.e., the subject sensed an arc with this outward curvature as being straight (Fig. 3C), and so would report a truly straight boundary as inwardly curved more than 50% of the time. The displacement (*d*) between the midpoint of the arc and a straight line (dotted-trace) is marked by the gray bar.

We also assessed subjects' haptic sensitivity, that is, how quickly the subject's response varied with changes in the boundary's curvature. To quantify sensitivity, we computed the difference in curvature between the 25% and 75% points (solid diamonds) on the function, as shown in Fig. 3B. This difference, which is inversely related to the steepness of the sensitivity function, is a measure of subjects' uncertainty: the steeper the function, the smaller the range of uncertainty and therefore the less the subjects' hand path has to curve before they are confident about the direction of curvature. In this example, the difference threshold was 0.68/m. The upper and lower curvatures of the threshold difference (the curvature at the 25% and 75% points) are drawn in Fig. 3C (dashed lines). The threshold difference reflects how much the virtual wall had to curve before subjects responded consistently that the wall curved in the concave direction versus the convex direction.

Subjects were adept at detecting small curvatures of their hand path across the different locations. Curvature

Table 1 Directional and absolute curvature-detection biases, and difference thresholds for all subjects, averaged across hand path location (SDs are shown *in parentheses*) for Experiment 1. Corresponding hand displacement at the mid-point of the arc from a straight path. Positive values: outward curves are convex in the 12 to 5 o'clock directions with respect to the subject

Subjects	Curvature bias		Absolute curvature bias		Curvature difference threshold	
	Mean (±SD) curvature (/m)	Hand displacement (cm)	Mean (± SD) curvature (/m)	Hand displacement (cm)	Mean (±SD) curvature range	Hand displacement (cm)
A	−0.31 (±0.40)	−0.087	0.40 (±0.30)	0.113	0.85 (±0.75)	0.239
B	0.12 (±0.57)	0.034	0.49 (±0.31)	0.138	0.82 (±0.42)	0.231
C	−0.15 (±0.69)	−0.042	0.61 (±0.36)	0.172	1.74 (±0.85)	0.488
D	−0.12 (±0.77)	−0.034	0.65 (±0.43)	0.183	1.28 (±0.60)	0.361
E	0.10 (±0.67)	0.028	0.55 (±0.38)	0.155	0.91 (±0.58)	0.257
F	0.04 (±0.80)	0.011	0.63 (±0.48)	0.177	1.07 (±0.55)	0.301
Mean	−0.05 (±0.65)	−0.040	0.56 (±0.38)	0.158	1.11 (±0.62)	0.310

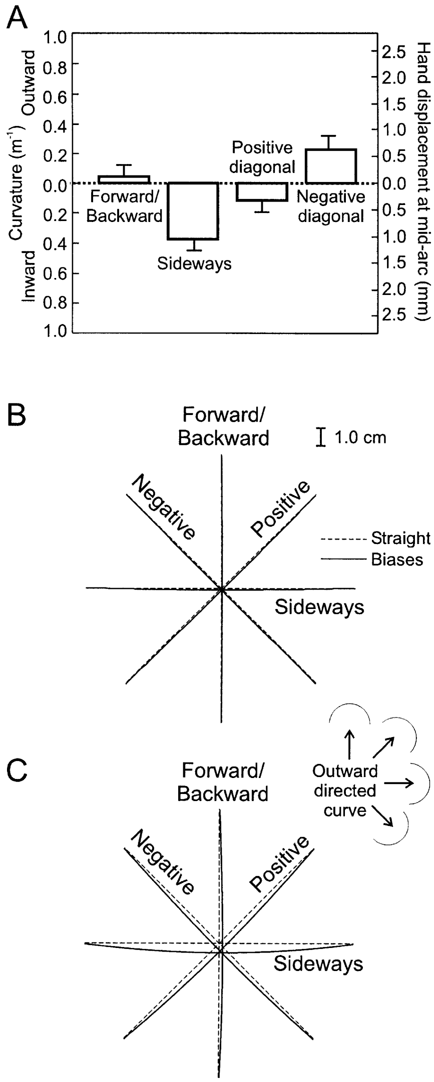

Fig. 4A–C Curvature-detection biases for the four hand path directions. **A** Bar graph showing the mean curvature (/m) and corresponding hand displacement at the mid-arc (d in Fig. 3C) of biases (averaged across locations and subjects) for the forward/ backward, sideways, positive- and negative-diagonal directions. *Error bars* are standard errors of the mean (SE), across locations for all subjects. **B** Arcs corresponding to these mean biases are drawn (*solid traces*) for each of the four directions. *Dashed traces* are veridical straight lines. Arcs are curved outward when they are convex in the 12 o'clock to 5 o'clock directions relative to the subject (see *inset*). **C** Same arcs as in **B** with curvatures magnified fivefold for better visibility

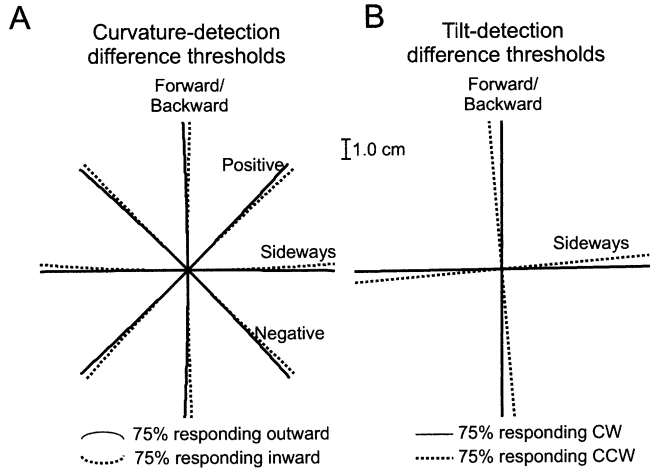

Fig. 5A, B Upper and lower difference thresholds as a function of hand path direction for the Curvature-detection and Tilt-detection experiments. *Solid and dotted traces* are the curved **(A)** and tilted **(B)** boundaries that subjects identified as outwardly curved and tilted CCW 75% and 25% of the time (average across locations and subjects). The difference between these paired arcs or tilted edges for each direction is the difference threshold

biases (averages across locations are listed in Table 1) were small, with a grand mean bias, averaged across the 48 hand path locations, of about 0.05/m (or a radius of 18.7 m). So, subjects, on average, tended to regard haptic boundaries curved 0.05/m inward as straight. These mean biases are small because positive and negative individual biases in different parts of the workspace tended to cancel out. The mean absolute bias was larger, with a curvature of 0.56/m or a radius of 1.80 m. These absolute biases (averages across locations are listed in Table 1) suggest that subjects were not perfectly veridical in their haptic perception of straightness, tending to perceive a hand path curved slightly outward or inward as 'straight'. But even these values correspond to very small curves; arcs with curvatures of 0.05 and 0.56/m would at their midpoints move the subject's hand only 0.04 cm and 0.16 cm away from a straight path 15 cm long.

Subjects' sense of curvature did not vary significantly with the location of the hand path (p>0.05), nor did the biases differ as a function of distance from the subject. Curvature biases for overlapping edges approached from different directions (seven in total; the inner grid in Fig. 1A) did not vary significantly either (p>0.05), with a mean difference of 0.18/m of curvature and a correlation of 0.64.

The direction of subjects' hand paths did affect what they perceived as haptically straight; curvature biases differed significantly for the four hand path directions (repeated one-way ANOVA, $F_{(7,245)}$=5.31, p<0.001). As shown in Fig. 4, subjects misjudged sideways boundaries the most, with a mean bias curvature of 0.38/m (radius of 2.7 m) curving inward across the different locations. Their judgments were more veridical for forward-backward movements, with a mean bias of 0.05/m (radius of 20.7 m) curved outward. For positive-diagonal hand paths, subjects had a mean inward-curved bias of 0.12/m, and for negative-diagonal paths a mean outward-curved bias of 0.23/m (outward curves are those that are convex toward the sector between 12 and 5 o'clock, as shown in the inset in Fig. 4). These mean biases are drawn to scale in Fig. 4B, and are drawn with fivefold-magnified curvature for better visibility in Fig. 4C.

Although subjects showed small curvature biases for hand paths traveling in different directions, the difference thresholds did not vary significantly across the locations

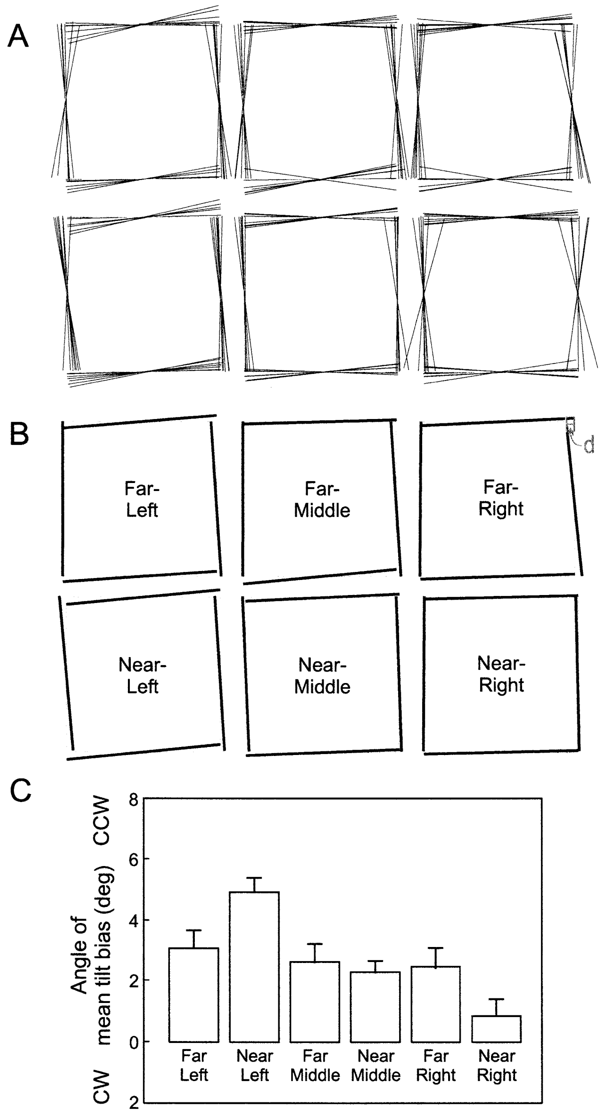

Fig. 6A–C Actual tilt biases for the 24 edges tested. For clarity, we have shifted the six squares apart so that their edges do not overlap. *Dark solid lines* are the tilt biases for each subject **(A)** and their mean **(B)**, while the *gray solid lines* are the veridical cardinal directions. The *gray error bar* in **B** (*right*) shows how far the hand was displaced (*d*) from the veridical cardinal direction at the corner of the tilt edge. **C** Bar graph showing the angle of mean tilt bias (in degrees) and SE (error bars) for the six workplace locations

or directions of the haptic boundaries (p>0.05). Subjects' mean difference thresholds across the 48 boundaries are listed in Table 1. Mean difference thresholds are shown in Fig. 5A, where we have drawn the arcs that subjects would perceive as curved outwards 25% and 75% of the time (dotted and solid traces, respectively) for each of the four directions; the difference threshold is represented by the difference between the two arcs. The grand mean difference threshold was 1.11/m of curvature (±0.62, mean standard deviation across boundaries), so that the subject's hand trajectory had to change its curvature by 1.11/m before the subject was 75% sure it curved outward compared to when he/she was equally certain that it was curved inward. In other words, subjects' hands would have to be displaced about 0.16 cm to either side of their bias arc before they would perceive the curvature to be in one direction 75% of the time.

Experiment 2: tilt direction

When we mapped our subjects' sensitivity in the Tilt experiment, we found that they were biased in their judgments of what they thought was a cardinal direction (all biases for each subject are drawn in Fig. 6A, with averages listed in Table 2). The tilt bias represents the angular direction of the hand path the subject would perceive equally likely as tilted CW or CCW away from the forward-backward or sideways cardinal direction. Subjects misperceived hand paths tilted on average 2.19° and 3.23° CCW as strictly in the forward-backwards or sideways direction. These mean tilt biases correspond to the hand being displaced 0.29 cm and 0.42 cm from the correct cardinal direction at endpoints of the virtual boundary (this displacement is indicated by a gray error bar in Fig. 6B).

Tilt biases varied significantly with the workspace location as indicated in Fig. 6, with the mean tilt biases shown in Fig. 6B ($F_{(5,135)}$=3.22, p<0.01). This variation was somewhat idiosyncratic, with two subjects misjudging cardinal directions as CCW more in the rightward workspace, while another two showed the reverse, greater CCW errors for boundaries to their left. The remaining three subjects showed errors of similar magnitude across

Table 2 Directional and absolute tilt biases, and difference thresholds for all subjects, averaged across hand path location (SDs are shown *in parentheses*) for Experiment 2. Corresponding hand displacement at the tips of the straight edge from the closest cardinal direction. Positive values: CCW-tilted edges

Subjects	Tilt bias		Absolute tilt bias		Tilt difference threshold	
	Mean angle (±SD) in degrees	Hand displacement (cm)	Mean angle (±SD) in degrees	Hand displacement (cm)	Mean angular range (±SD) in degrees	Hand displacement (cm)
A	1.03 (±2.85)	0.13	2.43 (±1.75)	0.32	3.26 (±1.75)	0.43
B	5.63 (±5.42)	0.74	7.03 (±3.30)	0.92	3.87 (±3.05)	0.51
C	1.53 (±5.61)	0.20	4.78 (±3.16)	0.63	6.42 (±3.44)	0.84
D	3.06 (±4.46)	0.40	3.78 (±3.87)	0.49	4.07 (±2.18)	0.61
G	−0.03 (±6.48)	0.00	4.89 (±4.13)	0.64	5.64 (±1.91)	0.86
H	2.88 (±2.34)	0.38	3.12 (±2.00)	0.41	4.62 (±2.38)	0.54
I	4.89 (±4.43)	0.64	5.54 (±4.13)	0.73	6.55 (±2.77)	0.74
Mean	2.71 (±4.51)	0.36	4.57 (±3.11)	0.59	4.92 (±2.50)	0.65

the workspace, and usually in the CCW direction. There was little variation along the far-near dimension: tilt biases were similar across far and near boundaries. Figure 6C shows that for boundaries forming the near-right square, subjects on average showed only a small tilt bias (mean of 0.86° CCW) in their perception of 'cardinal' directions, but in the near-left square they misperceived tilts on the order of 4.91° CCW (on average) to be cardinal.

Why did subjects' misestimates of hand path tilt vary across the workspace? Were subjects less haptically sensitive for boundaries farther from the body? The data do not support this idea: tilt biases did not vary as a function of the distance of the boundaries from the subject's torso, or from their right shoulder. But these biases for sideways hand motion did increase for more laterally located boundaries ($F_{(2,54)}$=5.09, p<0.01), with the smallest bias for hand movements made in front of the subjects' right shoulder (CCW tilts of 1.67°), and the largest biases for movements to the far left (4.94° CCW).

Subjects showed considerable uncertainty (Table 2): their hand paths had to tilt, on average, 4.92° (±2.50, mean standard deviation across boundaries) to take them from a path that they were 75% certain was directed CCW (dotted lines in Fig. 5B) to one they were 75% certain was directed CW (dashed lines), but these difference thresholds did not vary significantly with boundary location.

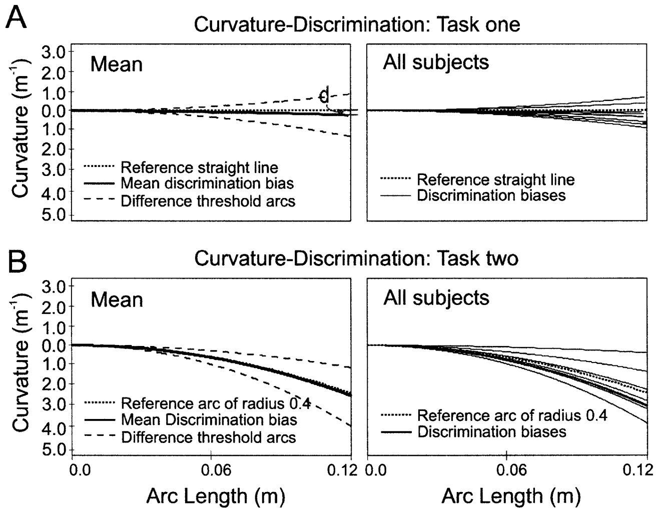

Fig. 7A, B Curvature-discrimination biases for arcs of different curvature. The *dotted traces* are the reference arcs: a straight line for task one **(A)** and an arc with a curvature of 2.5/m for task two **(B)**. *Left panels Solid traces* are the mean biases (across subjects), flanked by arcs with the curvatures at the 25% and 75% points (*dashed traces*) of the subjects' sensitivity function, representing the lower and upper difference thresholds. Arcs are drawn to scale, and in the curvature scales on the ordinates *tick marks* are placed where an arc of a given curvature would intersect the ordinate after a horizontal run (along the abscissa) of 12 cm. *Right panels Solid traces* are the curvature-discrimination biases for all subjects. For comparison, both the biases and the reference arcs are drawn running to the right rather than on opposite directions from the transition point as they did in the task

Experiment 3: curvature discrimination

Subjects performed well when asked to determine which side (left or right) of an arc had the greater curvature. When they were asked to differentiate a curve from a straight boundary, they reported that a boundary with an average curvature of 0.24/m felt as 'curved' as a straight one, as shown in Fig. 7A. Curvature-discrimination biases and their threshold differences for all subjects are listed in Table 3. The biases were somewhat smaller than those found when subjects were asked to detect whether the sideways arcs curved inward or outward in Experiment 1 (0.37/m) ($t_{(31)}$=3.381, p<0.01). However, the amount of average uncertainty was larger in the Curvature-discrimination experiment, with a curvature range of 2.26/m compared to 1.07/m for sideways hand paths in the Curvature-detection experiment.

Subjects were also able to notice small differences in curvature when judging between two arcs, one of which always had a curvature of 2.50/m. On average, the 50% threshold curvature was 2.61/m (or radius of 0.38 m), which is only 0.11/m more curved than the reference arc, as shown in Fig. 7B. Although this curvature-difference bias (listed in Table 3 for all subjects) seems smaller than that seen in the other Curvature-discrimination task when

Table 3 Curvature-discrimination biases and difference threshold for all subjects for the two tasks in Experiment 3. Values shown are the difference between the curvature of the bias and that of the reference arcs: a curvature of zero for task one, and a curvature of 2.50 for task two. Positive values: larger outward curvature relative to the reference arc

Subjects	Arc-comparison: task one		Arc-comparison: task two	
	Discrimination bias (/m)	Difference threshold (/m)	Discrimination bias (/m)	Difference threshold (/m)
A	0.93	1.11	0.37	1.18
B	−0.73	4.20	0.66	5.28
C	0.35	1.85	0.58	0.49
D	0.36	4.08	0.76	0.43
E	0.63	1.43	1.42	1.84
G	0.18	1.62	−2.10	3.71
J	−0.38	0.78	−0.18	3.38
K	0.76	2.58	0.62	6.26
L	0.09	2.72	−1.10	3.32
Mean	0.24	2.26	0.11	2.88

subjects compared an arc with a straight boundary, the biases were not significantly different, and the variability across subjects for this double-arc comparison (SD 1.14) was larger than in the other (SD 0.42). The amount of mean uncertainty was similar, 2.88, but again was much larger than that for Curvature-detection in Experiment 1.

Experiment 4: rate-of-change-of-curvature

In this experiment, subjects moved their hand along a quarter-circle of radius 0.10 m, and along a spiral where the radius of curvature could either increase or decrease. Most subjects mistook inward-spiraling paths (solid lines in Fig. 8, right panels) for portions of circles (dotted-trace). That is, their spiral biases were usually negative, as shown in Table 4. On average, subjects perceived as circular a spiral whose radius of curvature decreases by 0.041 cm for every centimeter of arc length (sp=0.041). Such a rate of decrease would bring the subject's hand 0.63 cm, or 6%, closer to the arc's center of rotation after 90° of spiral. While the spiral biases did not differ when the 90° arc of constant radius was to the left (Fig. 8, top row) of the subjects or to their right (Fig. 8, bottom row) (p>0.05), some spiral biases did vary with location: two subjects showed spiraling-out biases when the spiral arc was on the left side, but a different, third subject was the only one to show a spiraling-out bias when the spiral was on the right.

The degree of uncertainty was such that on average subjects felt a circular arc to be spiraling out 75% of the time, as can seen by overlapping dash-line arcs representing the outward difference threshold (Fig. 8, left panels) and the reference arcs with the constant radius (dotted arc). Likewise, the arc had to spiral in by an average of 9% in 90° before subjects were 75% sure that the hand path was spiraling in. The difference thresholds were the same when the 90° of constant radius was to the left (Fig. 8, top row) of the subjects or to their right (Fig. 8, bottom row).

Experiment 5: circularity

Subjects varied in how well they noticed changes in curvature when tracing an ellipse. When they were asked to select which of the two orthogonal axes of the ellipse

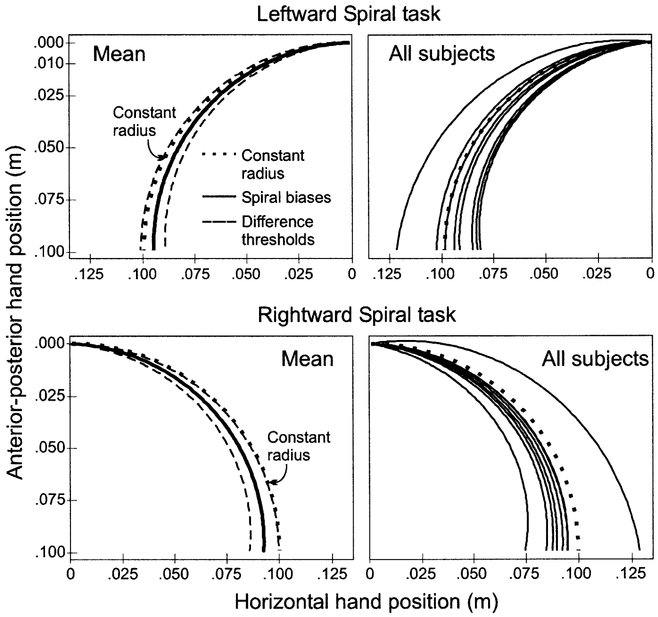

Fig. 8 Biases in the Spiral experiment. These biases are the 90° spirals (*solid lines*) that subjects reported as having a constant radius of curvature. *Top and bottom panels* show the results when the spiral was on the left and right sides, respectively. *Dotted traces* are the reference arcs with a constant 10-cm radius, which were located on the opposite side of the spiral during the task, but are drawn on the same side here for comparison. Spirals are drawn to scale, with the origin marking the beginning of the spiral. *Left panels Solid traces* are the mean biases, flanked by the mean spiral arcs at the 25% and 75% points (*dashed traces*) of subjects' sensitivity function. *Right panels* Biases for each subject

felt longer after tracing its circumference, some subjects showed large biases, and thus perceived elliptical traces to be perfectly circular. The subjects' perceived 'circles', and their average, are drawn as solid traces in Fig. 9A, B. The dotted traces are veridical circles. When tracing the larger ellipses (about 20 cm across), subjects perceived as circular, on average, ellipses that were actually 2% or 0.39 cm wider; with the smaller shapes (about 10 cm across) they perceived as circular ellipses that were 9%, or 0.83 cm, wider. Figure 9C shows the average range of uncertainty: for one ellipse typical subjects would be 75% certain that it was flatter than a circle; for the other they would be 75% certain that it was taller than a circle.

We also describe a subject's bias as the ratio of the actual sizes of the axes (width/length) that they thought

Table 4 Spiral biases and difference threshold for all subjects for the two tasks in Experiment 4. Values shown are the *sp* values for the logarithmic spiral equation (Eq. 1), which corresponds to the rate of change of radius relative to arc length. Positive values: spiral outward (Increase in radius)

Subjects	Leftward spiral		Rightward spiral	
	Bias	Difference threshold	Bias	Difference threshold
A	−0.123	0.050	−0.035	0.066
B	−0.095	0.073	0.165	0.082
C	−0.111	0.142	−0.071	0.234
G	0.133	0.099	−0.190	0.070
H	−0.049	0.061	−0.086	0.106
M	−0.002	0.086	−0.033	0.020
N	0.024	0.050	−0.108	0.102
Mean	−0.032	0.080	−0.051	0.097

Table 5 Circularity or axis-ratio biases and difference threshold for all subjects for the big and small circle tasks in Experiment 5. Positive values: circles are taller than they are wide

Subjects	Big elliptic traces		Small elliptic traces	
	Bias	Difference threshold	Bias	Difference threshold
A	0.992	0.108	1.001	0.134
B	0.931	0.197	0.797	0.127
C	1.289	0.086	1.076	0.083
E	0.998	0.228	0.955	0.125
G	0.800	0.101	0.889	0.090
K	1.044	0.159	0.975	0.302
L	0.866	0.158	0.763	0.065
O	0.934	0.164	0.827	0.072
P	0.971	0.303	0.967	0.517
Mean	0.980	0.167	0.917	0.168

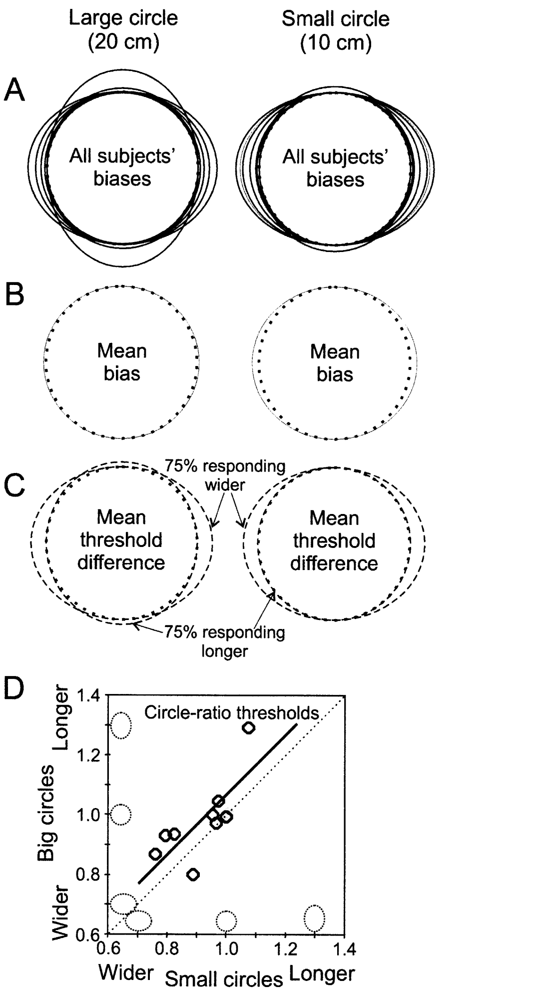

Fig. 9A–D Biases in a Circularity task where subjects judged which of an ellipse's two axes were bigger (circularity). *Solid traces* are ellipses perceived as circular by each subject (**A**) and their average (**B**). *Dotted traces* are veridical circles. **C** *Dashed traces* are ellipses that subjects would, on average, respond as feeling longer or wider than a circle 75% of the time. **D** Ratio of width-to-length of axes that subjects perceived to be equally long. Ratios for larger circles are plotted as a function of ratios for smaller circles. *Solid line* is the slope of best fit

were of equal size, so that a ratio of 1.0 represents a perfect circle, a ratio below 1.0 a wide/short circle, and a ratio above 1.0 a long/skinny circle (Table 5). To determine whether subjects' haptic performance changed with circle size, we plotted the ratios for the larger circle task as a function of those for the smaller circle task for each subject in Fig. 9D. Subjects' haptic misperception of what felt to be perfectly circular did not differ much for the two sizes as shown by the similar mean circular thresholds in Fig. 9B and by the near-unity fit (slope of 1.14) between the axes-ratio thresholds for large and small traces.

Discussion

In this study, we assessed people's ability to sense various geometric features with their unseen hand movements across a horizontal workspace using their haptic sense of arm motion. We found that subjects were quite good at detecting the absolute curvature of the hand path, but that their judgments were less veridical and showed some anisotropy when estimating the 'cardinal' direction of the hand path. While subjects did well at discriminating between differences in the curvature of their hand paths, they were somewhat less accurate at judging these relative curvatures than they were at detecting absolute curvature. Subjects also tended to misestimate how much their hand path curved along the mediolateral direction when detecting changes in the radius of curvature and distortions in the shape of the circular hand traces.

Detecting curvature of the hand path

Subjects were surprisingly good at detecting the curvature of their hand path. Their biases, or what they would regard as indistinguishable from a straight edge, corresponded roughly to radii between 3.2 and 25.6 m, or an absolute peak displacement of the hand (at the midpoint of the arc) from a straight line of about 0.04 cm on average. Such a small mean hand displacement represents only 0.27% of the 15-cm distance along the arc's base (arc-base displacement). Curvature biases were so slight that the arcs drawn from subjects' mean biases for the different hand path directions in Fig. 4B can hardly be seen to curve at all. Thus, subjects' internal estimates of haptic 'straightness' were near veridical.

These curvature biases were much subtler than the curves seen in natural reaching movements (Atkeson and Hollerbach 1985; Wolpert et al. 1994; Miall and Haggard 1995; Klein Breteler et al. 1998; Boessenkool et al. 1998; Sergio and Scott 1998; Jackson and Newport 2001), which show arc-base displacements of 1–3%. These haptic biases were also comparable to *visual*-curvature biases found when people judged the direction of curvature of a cursor spot moving along a 40-cm trajectory in a horizontal plane (Wolpert et al. 1994).

Although our subjects received no tactile stimulation beyond the pressure on the palm of the hand and the fingers as they gripped and moved the manipulandum along the virtual boundary, their curvature biases resembled those in other curvature-detection studies where subjects used their fingers to trace a real curved strip in the horizontal plane. For instance, our subjects displaced their hand from a 'straight' line by just 0.70% of the base length for sideways boundaries and 0.09% for forward-backward boundaries, while subjects tracing a 50-cm-long curved edge with their index finger showed biases with average arc-base displacements of 0.72% and 1.06% for sideways and forward-backward arcs, respectively (Miall and Haggard 1995). When subjects traced a 20-cm-long sideways curved strip with one or two fingers, their mean curvature bias had an arc-base displacement of 0.74% when their elbow was stationary and 0.12% when the arm was fully extended (Davidson 1972). A similar displacement of 0.45% was found when subjects reported the curvature direction of small arcs, 2, 3 and 4 cm in length, after tracing them with their middle finger (Gordon and Morison 1982). Although their review of the then-existing literature led Gordon and Morison to conclude that people are more sensitive to curvature when scanning movements are small and do not involve movement of the forearm, our results do not support this. People are just as sensitive to curvature displacement when scanning involved moving the whole arm (as in our experiment) as they were when tracing the curved edge with their finger.

Subject's sensitivity to curvature did not vary with the location of their hand path, or with the distance of their hand movements from their shoulder or torso. Thus, the proximity of the hand to the body or the degree by which the arm was extended did not influence what subjects perceived as 'straight'. However, subjects' perception of straightness did vary significantly with the direction of their hand path. On average, subjects perceived a sideways arc that displaced their hand 0.11 cm toward themselves (at the arc's midpoint) as non-curving. This displacement is about 10 times larger than the hand shift (to the left) that subjects would regard as straight in the forward-backward direction and 2–3 times larger than those for the diagonally directed hand paths (Fig. 4). The subjects in the study by Miall and Haggard (1995) described above showed a different pattern of biases as a function of arc direction, with larger biases for curved edges oriented diagonally, and a smaller one for sideways-oriented arcs. These biases were slightly bigger than ours but varied less with direction (whether this variation was significant was not reported). In this same study, and in one by Van Thiel et al. (1998), subjects' hand paths during point-to-point movements showed a pattern of direction-dependent curvature similar to our experiments. This suggests that our subjects' curvature biases, derived from haptic information about arm motion, are similar to curvatures found during natural arm movements, albeit smaller. Because this direction-dependent sensitivity to hand path curvature was uniform across the workspace, this effect is likely not due to variations in the motion at the shoulder and elbow joints. Instead, these direction-dependent biases may be due partly to variability in our internal representation of direction within the workspace.

Subjects were very confident about even relatively slight curvature. Typically, they responded 'out' for an outward curvature of 0.50/m (hand displacement of 0.14 cm) and 'in' for an inward curvature of 0.61/m (hand displacement of 0.18 cm) 75% of the time. So, on average, a subject's hand needed to be displaced by only 0.31 cm (difference threshold of 1.11/m) before they were confident in the curvature direction of the hand path.

Acuity of our haptic sense of arm movement

Sensory information about arm movement is derived from both tactile sensation through the skin and kinesthetic sensation of the position and movement of the joints and muscles of the hand and arm. From these signals, humans are able to match the joint angles of one arm with those of the other arm with a precision in the range of 1–7° (Soechting 1982; Clark et al. 1995), detect changes in the joint angles of the knee by 3–4° (Horch et al. 1975), discriminate changes in haptically felt angles for shoulder rotations of 0.5° (Voisin et al. 2002a), and localize a proprioceptive target with a joint angle precision of 0.6–1.1° (van Beers et al. 1998). Humans are also more accurate at locating or reproducing arm positions after they have actively moved their arm than when the arm is moved by the experimenter or when their arm remains stationary for a period of time (for review, see McCloskey 1978). Our study suggests that the haptic sense of arm motion is more acute than previously appreciated. Although we did not measure the joint positions of the arm during the experiments (since pilot data showed that haptic perception was uniform across different arm configurations), we can estimate how accurately subjects perceived slight rotations of their shoulder joints when reporting curvature direction. From our measurements of arm segment length, we estimate that in the two cases – when subjects were sure the arc curved out and when they were sure it curved in – their shoulder angles differed by just 0.2° at the mid-arc. Thus our results show subjects can discriminate postural changes about an order of magnitude smaller than those reported in proprioceptive matching tasks, consistent with the notion that haptic sense integrates information from several sensory modalities.

Indeed, haptic perception is also influenced by tactile pressure on the hand as subjects gripped and moved the manipulandum. Subjects were better able to localize their hand position after touching an object (such as a table) following an arm movement than when they moved the arm without making any tactile contact (Tillery et al. 1994; Lackner and DiZio 1998). Likewise, Voisin et al. (2002b) found that removing cutaneous input by anesthetizing the fingers impaired haptic perception. The same subjects performed even more poorly when the cutaneous feedback was provided but kinesthetic information was removed. Thus, pushing against the manipulandum contributed to our subjects' sensitivity to the curvature or direction of the hand path more than if they had merely moved along the same trajectory empty-handed.

Detecting the tilt of the hand path from the cardinal direction

Subjects' internal estimates of the cardinal directions in the horizontal plane were less veridical. They made larger and more consistent errors, averaging between 2 and 3° CCW, when estimating how their hand paths tilted relative to a purely sideways or forward-backward direction. Although these tilt biases were significant, they were smaller than biases reported for tasks where subjects had to match rotating bars with reference bars oriented in the cardinal directions in the horizontal plane (Kappers and Koenderink 1999; Kappers 1999) and the frontal plane (Appelle and Gravetter 1985). This suggests that people may be better at judging when their hand path deviates from their internal sense of the cardinal directions than they are at judging the spatial relations between felt objects.

In the other tilt detection study using a similar manipulandum (Fasse et al. 2000), subjects showed a 5° CCW bias for a forward-backward hand path, but a 5° CW bias for a sideways hand path. The difference in our results for sideways hand paths may be due to a difference in hand path length: 15 cm in our study versus 5 cm in theirs. Also, their subjects were tested on boundaries radiating from a common hub at angles of 5° from 90° left to 90° right, which may have put a ceiling on their judgments for sideways-oriented boundaries since they were not tested on CCW tilts from the sideways direction.

Comparing detection sensitivity of hand path curvature versus hand path direction

Subjects were less sensitive to tilts in their hand paths compared to curvature. When expressed as displacements of the hand from a non-curving or cardinal pathway, biases and difference thresholds for tilt judgments were several times larger than those for curvature (see Tables 1, 2, Fig. 5). A similar result has been found in a visual hyperacuity task: subjects were better at detecting curvature than tilt in flashed visual stimuli (20 arc min high) when the curvature and tilt caused equal horizontal deviations from a straight line (Fahle 1997). Why are we more sensitive to curvature than tilt? Since we are constantly moving with respect to objects (or they move with respect to us), we may be only seldom or briefly exposed to objects that are strictly cardinally oriented relative to us. Thus, we may be more sensitive to, or better recognize, objects with geometric features like curvature that are more constant and do not change so much with perspective. The lack of correlation between biases for curvature-detection and those for tilt-detection, even for similar hand path locations, also suggests that the haptic sensitivities to these two features develop independently.

Finally, the perception of haptic space was relatively uniform throughout the workspace. Hand paths farther from the body or right shoulder did not affect biases or uncertainty for tilt and curvature estimates. While some spatial anisotropies have been seen for haptic discriminations of stimulus size (an effect known as the tangential-radial illusion; Hogan et al. 1990; Armstrong and Marks 1999), no one has previously reported how haptic perception for other geometric features varies across the workspace. Because of the location independence of haptic perception in our first two experiments, we performed the remaining three only in the central workspace.

Curvature discrimination

Subjects were relatively good at discerning which of two arcs had the greater curvature. They were equally sensitive to differences in curvatures when comparing a straight and a curved boundary and when comparing two curved boundaries, with similar curvature-discrimination biases and difference thresholds for the two tasks.

By looking at the curvature-discrimination results for Task 1 of Experiment 3 and the curvature-*detection* results for sideways arcs in Experiment 1, we can compare relative judgments with absolute judgments of curvature. While the biases appeared to be smaller for curvature discrimination than detection, the range of uncertainty was twice as large for discrimination as for detection (2.26/m vs 1.07/m). This means that for a typical subject, a path whose curvature is, say, 0.8/m (that is, greater than the typical bias plus 1.07 over two, but less than bias plus 2.26 over two) will appear curved rather than straight, but will not be clearly more curved than a straight reference path. The paths were shorter in the curvature-discrimination task (12 cm vs 15 cm), but it is unlikely that this difference in length can explain the difference in performance in the two experiments as the arc-base displacement ratio was still much larger for curvature discrimination than detection (2.1% vs 0.3%), and, as described in the following paragraph, this arc-base ratio is constant across base lengths ranging between 4 and 90 cm for curvature discrimination (Louw et al. 2000).

The results of our Curvature-discrimination experiment, comparing a straight boundary with a curved one, were consistent with values found by Louw et al. (2000), who reported a linear relationship between the 75% threshold and stimulus length. This is so despite the large differences in our two experiments; in their study, subjects kept their elbow stationary and used two fingers to trace along a vertical, curved foam strip. Again, this suggests that haptic sense of motion along a virtual wall is as good as finger-tactile signals at providing accurate information about relative curvature.

Detecting rate of change of curvature

Subjects mistook inward-spiraling hand paths (with decreasing radius of curvature) for circles. While our study is the first to measure haptic sensitivity to the rate of change of curvature, the results were consistent with the sensitivities our subjects showed for other geometric features in Experiments 3 and 5. That is, our subjects required about the same hand displacements to be confident that a path was spiraling in rather than out as they did to feel sure that one arc was more curved than another, or that an ellipse was flatter rather than taller than a circle. In all three of these experiments, subjects' biases and difference thresholds were similar and relatively small.

Detecting distortions in circularity

Subjects also appeared to underestimate change in curvature along the mediolateral direction when judging whether the ellipse they traced was wider or longer (larger in the sideways or forward-backward direction). This distortion in subjects' perception of circularity is similar to the well-known tendency for people to perceive a square's horizontal width as feeling smaller than its height: the tangential-radial (or horizontal-vertical) illusion (Hogan et al. 1990; Armstrong and Marks 1999). Subjects in our study were required to feel the perimeter of the ellipse rather than moving along its axes, so these results are not purely an estimate of width and length of the axes. But in other studies, subjects showed similar biases in circularity when drawing circles with their unseen hand: they tended to generate wide ellipses when asked to draw circles 10 cm in diameter with a manipulandum (Fasse et al. 2000) or circles 16 cm in diameter with a pen on a digitizing table (Verschueren et al. 1999).

Conclusion

Haptic perception is a complex process but ultimately it relies in large part on geometric information provided by kinesthetic and tactile cues. We found that humans are very sensitive to absolute curvature, but less so for the direction of straight lines or differences in curvature of two contours. We also found biases in haptic judgments of these quantities and of the rate of change of curvature. By quantifying these effects, our experiments help assess the acuity of haptic perception. It is tempting to suggest that objects having complex shapes are identified haptically and reconstructed (Shimansky et al. 1997) by identifying the geometric properties of segments, and the relations among them. If so, our experiments have helped to define the limits of acuity in this process.

Acknowledgements We thank Ms. Obianuju Obi for her assistance in the experiments and Dr. Martha Flanders for her helpful suggestions throughout the course of this investigation.

References

Appelle S, Gravetter F (1985) Effect of modality-specific experience on visual and haptic judgment of orientation. Perception 14:763–773

Armstrong L, Marks LE (1999) Haptic perception of linear extent. Percept Psychophys 61:1211–1226

Atkeson CG, Hollerbach JM (1985) Kinematic features of unrestrained vertical arm movements. J Neurosci 5:2318–2330

Boessenkool JJ, Nijhof EJ, Erkelens CJ (1998) A comparison of curvatures of left and right hand movements in a simple pointing task. Exp Brain Res 120:369–376

Clark FJ, Larwood KJ, Davis ME, Deffenbacher KA (1995) A metric for assessing acuity in positioning joints and limbs. Exp Brain Res 107:73–79

Davidson PW (1972) Haptic judgments of curvature by blind and sighted humans. J Exp Psychol 93:43–55

Fahle M (1997) Specificity of learning curvature, orientation, and vernier discriminations. Vision Res 37:1885–1895

Fasse ED, Hogan N, Kay BA, Mussa-Ivaldi FA (2000) Haptic interaction with virtual objects. Spatial perception and motor control. Biol Cybern 82:69–83

Gentaz E, Luyat M, Cian C, Hatwell Y, Barraud PA, Raphel C (2001) The reproduction of vertical and oblique orientations in the visual, haptic, and somato-vestibular systems. Q J Exp Psychol A 54:513–526

Gordon IE, Morison V (1982) The haptic perception of curvature. Percept Psychophys 31:446–450

Hogan N, Kay BA, Fasse ED, Mussa-Ivaldi FA (1990) Haptic illusions: experiments on human manipulation and perception of "virtual objects". Cold Spring Harb Symp Quant Biol 55:925–931

Horch KW, Clark FJ, Burgess PR (1975) Awareness of knee joint angle under static conditions. J Neurophysiol 38:1436–1447

Jackson SR, Newport R (2001) Prism adaptation produces neglect-like patterns of hand path curvature in healthy adults. Neuropsychologia 39:810–814

Kappers AM (1999) Large systematic deviations in the haptic perception of parallelity. Perception 28:1001–1012

Kappers AM (2002) Haptic perception of parallelity in the midsagittal plane. Acta Psychol (Amst) 109:25–40

Kappers AM, Koenderink JJ (1999) Haptic perception of spatial relations. Perception 28:781–795

Kesten H (1958) Accelerated stochastic approximation. Ann Math Stat 29:41–59

Klein Breteler MD, Meulenbroek RG, Gielen SC (1998) Geometric features of workspace and joint-space paths of 3D reaching movements. Acta Psychol (Amst) 100:37–53

Lackner JR, DiZio P (1998) Adaptation in a rotating artificial gravity environment. Brain Res Brain Res Rev 28:194–202

Louw S, Kappers AM, Koenderink JJ (2000) Haptic detection thresholds of Gaussian profiles over the whole range of spatial scales. Exp Brain Res 132:369–374

McCloskey DI (1978) Kinesthetic sensibility. Physiol Rev 58:763–820

Miall RC, Haggard PN (1995) The curvature of human arm movements in the absence of visual experience. Exp Brain Res 103:421–428

Pont SC, Kappers AM, Koenderink JJ (1998) The influence of stimulus tilt on haptic curvature matching and discrimination by dynamic touch. Perception 27:869–880

Sergio LE, Scott SH (1998) Hand and joint paths during reaching movements with and without vision. Exp Brain Res 122:157–164

Shadmehr R, Mussa-Ivaldi FA, Bizzi E (1993) Postural force fields of the human arm and their role in generating multijoint movements. J Neurosci 13:45–62

Shimansky Y, Saling M, Wunderlich DA, Bracha V, Stelmach GE, Bloedel JR (1997) Impaired capacity of cerebellar patients to perceive and learn two-dimensional shapes based on kinesthetic cues. Learn Mem 4:36–48

Soechting JF (1982) Does position sense at the elbow reflect a sense of elbow joint angle or one of limb orientation? Brain Res 248:392–395

Tillery SI, Flanders M, Soechting JF (1994) Errors in kinesthetic transformations for hand apposition. Neuroreport 6:177–181

Treutwein B (1995) Adaptive psychophysical procedures. Vision Res 35:2503–2522

van Beers RJ, Sittig AC, Denier van der Gon JJ (1998) The precision of proprioceptive position sense. Exp Brain Res 122:367–377

van Thiel E, Meulenbroek RG, Hulstijn W (1998) Path curvature in workspace and in joint space: evidence for coexisting coordinative rules in aiming. Motor Control 2:331–351

Verschueren SM, Swinnen SP, Cordo PJ, Dounskaia NV (1999) Proprioceptive control of multijoint movement: unimanual circle drawing. Exp Brain Res 127:171–181

Voisin J, Benoit G, Chapman CE (2002a) Haptic discrimination of object shape in humans: two-dimensional angle discrimination. Exp Brain Res 145:239–250

Voisin J, Lamarre Y, Chapman CE (2002b) Haptic discrimination of object shape in humans: contribution of cutaneous and proprioceptive inputs. Exp Brain Res 145:251–260

Wolpert DM, Ghahramani Z, Jordan MI (1994) Perceptual distortion contributes to the curvature of human reaching movements. Exp Brain Res 98:153–156